Part II

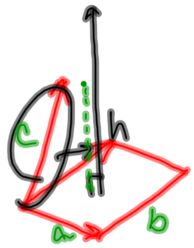
7. Find the volume of a parallelepiped defined by the following vectors. (10 pts)

$$\langle 2,3,5\rangle, \langle -1,3,0\rangle, \langle 5,8,0\rangle$$

Parallelepiped: a three-dimensional figure formed by six parallelograms

$$|a\times b| = |\langle -15, -5, 9\rangle|$$

$$= \sqrt{225+25+81} = \sqrt{331}$$

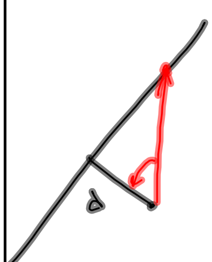


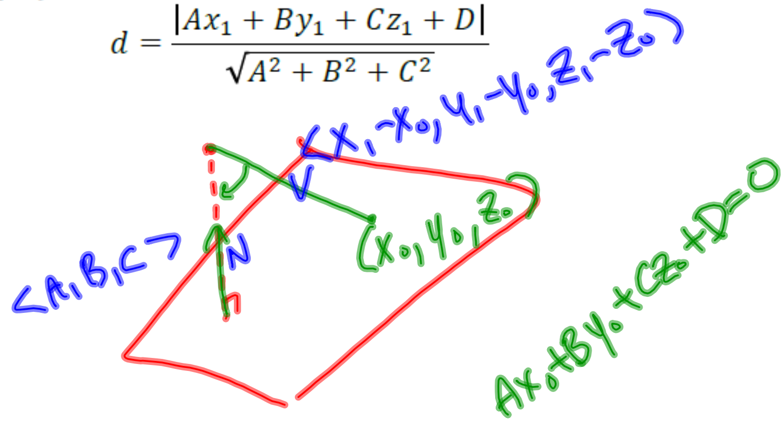
$$\text{Proj}_h\, c = \frac{h\cdot c}{|h|}\cancel{\frac{h}{|h|}} = \left|\frac{-75-40+0}{\sqrt{331}}\right| = \frac{115}{\sqrt{331}}$$

$$\boxed{115}$$

8. Show that the distance between a point (x_1, y_1, z_1), and a plane $Ax + By + Cz + D = 0$ is (5 pts)

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$





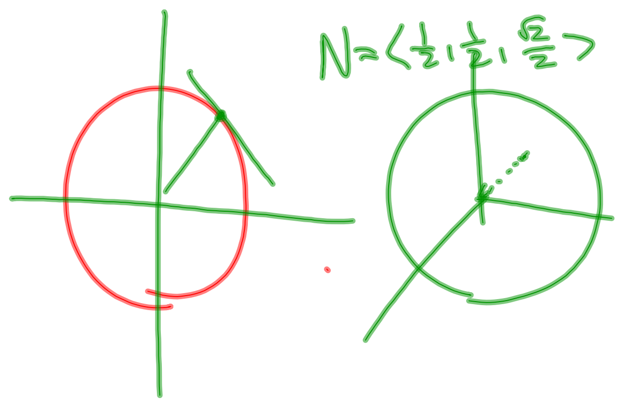
$$\text{Proj}_N V = \frac{|N\cdot V|}{|N|} = \frac{|Ax_1+By_1+Cz_1+D\ \cancel{-Ax_0-By_0-Cz_0}|}{\sqrt{A^2+B^2+C^2}}$$

E12

Eq. of Tangent plane

$x^2+y^2+z^2=1$ @ $\langle \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2} \rangle$

$N=\langle \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2} \rangle$



$\langle \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2} \rangle \cdot \langle x-\frac{1}{2}, y-\frac{1}{2}, z-\frac{\sqrt{2}}{2} \rangle = 0$

$\frac{1}{2}x+\frac{1}{2}y+\frac{\sqrt{2}}{2}z=1$

$\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2}=0$

$\sqrt{(x-0)^2+(y-0)^2}<\delta$

Def. $\delta=\frac{\varepsilon}{2}$

Assume $\sqrt{x^2+y^2}<\delta$

we need to prove $\left|\frac{2xy^2}{x^2+y^2}\right|<\varepsilon$

$\left|\frac{2xy^2}{x^2+y^2}-0\right|<\varepsilon$

$=\frac{2y^2}{x^2+y^2}|x| \le 2|x|$

$=2\sqrt{x^2} \le 2\sqrt{x^2+y^2}<\varepsilon$

$\sqrt{x^2+y^2}<\frac{\varepsilon}{2}$

$\sqrt{x^2+y^2}<\delta$

$2|x|=2\sqrt{x^2} \le 2\sqrt{x^2+y^2}<2\delta=\varepsilon$

$\frac{y^2}{x^2+y^2}2|x|<\varepsilon$

$\left|\frac{2xy^2}{x^2+y^2}\right|<\varepsilon$

$\therefore \lim = 0$